Selection of Secondary Measurement for Parallel Cascade Control

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Selection of secondary measurement is an important problem in chemical process control since important process units are multistage types or distributed parameter systems. Brosilow and coworkers (Weber and Brosilow, 1972; Joseph and Brosilow, 1978) proposed a method to select secondary measurements for inferential control based on the condition number of the selected secondary variables. Downs (Downs and Moore, 1981) used the method of singular value decomposition (SVD) to select temperature measurement locations in an azeotropic distillation column. Instead of using the condition number, the major component in the principle output direction is used in Downs approach. Furthermore, the SVD method offers a one-shot solution for selecting a few variables from many possible choices. Both approaches deal with the process transfer functions only (without considering load disturbances). Luyben viewed the measurement selection problem differently. The "Luyben's procedure" is to select a secondary measurement in such a way that when such particular secondary variables as a tray temperature is held constant, primary variables like product composition will have smallest offset in the face of load disturbance. Luyben's procedure was applied successfully to multicomponent distillation columns (Yu and Luyben, 1984).

All of the methods mentioned above deal with situations when the primary measurement is not available. Recent advances in analytical instruments make the on-line composition (most primary variables in chemical processes are compositions) measurement possible. In most cases, the dynamic response of a secondary measurement, e.g., a tray temperature, is faster than that of a primary measurement, e.g., composition. The selection of secondary measurement with the *presence* of primary measurement is a valid problem, since the faster secondary measurement can be used to improve control quality.

One way to combine the primary and secondary measurements in the control application is the parallel cascade control structure (Yu, 1988). The objective is to achieve good disturbance–rejection for *a* specific load change. Hsu et al. (1990) interpreted the parallel cascade control structure as an indirect feedforward control, since the secondary controller acts effectively as a feedforward controller for specific load changes. Unfortunately, most process units face more than one-load disturbances. The purpose of this note is to investigate the performance deterioration of the parallel cascade control when the real load disturbance is different from the nominal one (the load disturbance where the design of secondary controller is based on). Consequently, a measurement selection criterion is proposed to achieve best averaging disturbance–rejection. A depropanizer example is used to illustrate that control quality can be improved significantly with appropriate selection of secondary measurement.

Indirect Feedforward Control

A simple combination of fast secondary measurement (X_s) and slow primary measurement (X_p) is the parallel cascade control structure (Figure 1). The block diagram shows the closed-loop relationship between outputs (X_p and X_s), manipulated variable (m) and a specific load variable (L) via process (G_p and G_s) and load (GL_p and GL_s) transfer functions.The control objective is to maintain the primary output at set point by changing m. The closed–loop relationship for primary output is:

$$X_p = \frac{K_s K_p G_p}{1 + K_s(G_s + K_p G_p)} X_p^{\text{set}} + \frac{GL_p + K_s(GL_p G_s - GL_s G_p)}{1 + K_s(G_s + K_p G_p)} L \quad (1)$$

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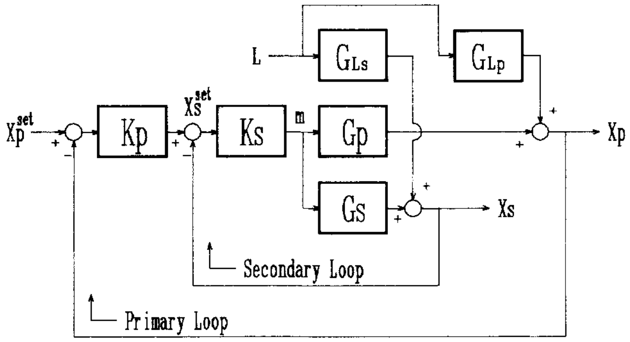

Figure 1. Indirect feedforward control under the parallel cascade control structure.

where K_p and K_s stand for primary and secondary controllers, respectively (Figure 1). As shown by Yu (1988), if the secondary controller is designed as:

$$K_s = \frac{1}{\left[\dfrac{GL_{s(0)}G_{p(0)}}{GL_{p(0)}G_{s(0)}} - 1\right]G_{s(0)}} \tag{2}$$

Perfect disturbance–rejection in the sense of integrated error (IE), $IE = 0$, can be achieved for the load disturbance (L). The reason is: the numerator of the second term on the righthand side of Eq. 1 is zero for $s = 0$. Parallel cascade structure with the secondary controller, K_s, designed according to Eq. 2 can be interpreted as indirect feedforward control, since the load disturbance, L, is sensed by the fast secondary measurement and the desired change in the manipulated variable, m, is made via the secondary controller in a feedback loop(Hsu et al., 1990). Despite the ability to handle a specific disturbance perfectly, i.e., $IE = 0$, there is a realisitic question to ask: what is the performance of the indirect feedforward control when the load disturbance is different from the nominal load disturbance. The interaction measure γ defined by Yu (1988) provides quantitative information in this regard.

For a chosen secondary variable and a specified load variable L, γ is defined as:

$$\gamma \equiv \frac{\left[\dfrac{\partial m}{\partial L}\right]_{X_s}}{\left[\dfrac{\partial m}{\partial L}\right]_{X_p}} \tag{3}$$

After some algebraic manipulation(Yu, 1988), we have

$$\gamma = \frac{GL_{s(0)}G_{p(0)}}{GL_{p(0)}G_{s(0)}} \tag{4}$$

Since only steady–state aspect is considered, the subscript (0) is dropped for clarity.

To test the effectiveness of parallel cascade control under uncertainties in load changes, comparison is made with respect to direct control of the primary variable, i.e., single-loop control of X_p. Consider the nominal load variable $\tilde{L}$ and its corresponding load transfer function $\tilde{G}L_p$ and $\tilde{G}L_s$. The nominal interaction measure $\tilde{\gamma}$ becomes:

$$\tilde{\gamma} = \frac{\tilde{G}L_s G_p}{\tilde{G}L_p G_s} \tag{5}$$

Note that since the uncertainty considered here is load variable, possible deviations in the definition of γ are in GL_p and GL_s. If the secondary controller is designed according to $\tilde{\gamma}$, we have

$$K_s = \frac{1}{(\tilde{\gamma} - 1)G_s} \tag{6}$$

Table 1. Process and Load Transfer Functions for the Primary (Xd) and Secondary (T_i) Outputs

Output	Manip. Variable	Loads Variable: Propylene	n-Propane	Isobutane	1-Butene	n-Butane
T_1	$\frac{-26.366}{6.0s+1}$	$\frac{-14.831}{7.8s+1}$	$\frac{-21.508}{7.8s+1}$	$\frac{-8.257}{4.8s+1}$	$\frac{-7.385}{5.4s+1}$	$\frac{-7.461}{9.6s+1}$
T_7	$\frac{-21.564}{4.2s+1}$	$\frac{-21.101}{7.8s+1}$	$\frac{-24.554}{4.8s+1}$	$\frac{-2.532}{4.8s+1}$	$\frac{-4.308}{9.0s+1}$	$\frac{-5.687}{12s+1}$
T_8	$\frac{-18.406}{6.0s+1}$	$\frac{-18.674}{4.5s+1}$	$\frac{-20.585}{6.0s+1}$	$\frac{-0.550}{9.0s+1}$	$\frac{-2.154}{12s+1}$	$\frac{-3.339}{18s+1}$
T_{13}	$\frac{-22.550}{5.0s+1}$	$\frac{-22.247}{5.0s+1}$	$\frac{-24.923}{9.0s+1}$	$\frac{-2.037}{10s+1}$	$\frac{-11.385}{10s+1}$	$\frac{-15.757}{12s+1}$
T_{14}	$\frac{-16.877}{3.6s+1}$	$\frac{-18.000}{7.2s+1}$	$\frac{-18.462}{9.6s+1}$	$\frac{-1.761}{4.8s+1}$	$\frac{-9.962}{9.0s+1}$	$\frac{-13.096}{10.8s+1}$
T_{15}	$\frac{-12.118}{6.0s+1}$	$\frac{-13.685}{7.2s+1}$	$\frac{-12.185}{9.6s+1}$	$\frac{-1.431}{7.8s+1}$	$\frac{-7.962}{9.6s+1}$	$\frac{-9.965}{10.8s+1}$
T_{20}	$\frac{-2.858}{19.5s+1}$	$\frac{-3.91}{21s+1}$	$\frac{-1.477}{36s+1}$	$\frac{-0.55}{36s+1}$	$\frac{-1.231}{15s+1}$	$\frac{-1.565}{18s+1}$
Xd	$\frac{-0.00596}{7.8s+1}$	$\frac{-0.00634}{19.2s+1}$	$\frac{-0.00812}{25.2s+1}$	$\frac{-0.00066}{12s+1}$	$\frac{-0.00585}{21s+1}$	$\frac{-0.00657}{22.8s+1}$

Table 2. Interaction Measurement Array

Tray No.	Loads Variable				
	Propylene	n-Propane	Isobutane	1-Butene	n-Butane
1	0.52879	0.59875	2.82801	0.28536	0.25671
7	0.91988	0.83576	1.06032	0.20653	0.23924
8	0.95375	0.82088	0.26984	0.11923	0.16457
13	0.92743	0.81123	0.81573	0.51327	0.63388
14	1.00262	0.80292	0.94225	0.58507	0.70392
15	1.06162	0.73775	1.06638	0.66939	0.74598
20	1.28609	0.37932	1.73781	0.43882	0.49674

For the nominal load disturbance, the IE of parallel cascade control(IE_{PC}) is zero for cases with $\tilde{\gamma} > 1$.

$$\frac{IE_{PC}}{IE_{SL}} = 0 \tag{7}$$

where IE_{SL} denotes the IE of the single-loop control. When the load disturbance is other than the nominal one, performance of the cascade control becomes (see Appendix A for derivation):

$$\frac{IE_{PC}}{IE_{SL}} = \frac{\tilde{\gamma} - \gamma}{\tilde{\gamma}} \tag{8}$$

where γ is the interaction measure for the *real* load disturbance. Equation 8 quantifies the performance of parallel cascade control with respect to uncertainties in load changes. Since such multiple load disturbances as feed compositions and feed flow rate disturbances in a multicomponent distillation column exist generally for most process units, Eq 8 can be used as a criterion for the selection of secondary measurement for parallel cascade control.

Selection of Secondary Measurement

For a system with m possible secondary measurements and facing n load disturbances, an $m \times n$ Γ array can be constructed with the entry γ_{ij} where i stands for the ith measurement location and j stands for the jth load disturbance.

$$\Gamma = \begin{bmatrix} \gamma_{11} & \cdots & \gamma_{1n} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \gamma_{m1} & \cdots & \gamma_{mn} \end{bmatrix} \tag{9}$$

An averaging integrated error ($\overline{IE}_i^{(k)}$) can be defined for the ith measurement location when the kth load disturbance is chosen as the nominal one.

$$\overline{IE}_i^{(k)} \equiv \frac{\sum_{j=1}^{n} \left| \left[\frac{IE_{PC}}{IE_{SL}}\right]_j^{(k)} \right|}{n} = \frac{\sum_{j=1}^{n} \left| \frac{\gamma_{ik} - \gamma_{ij}}{\gamma_{ik}} \right|}{n} \tag{10}$$

Equation 10 weighs each load disturbance equally. If one load disturbance is more important (e.g., more frequent occurrence) than the other, weighting factors can be applied to Eq. 10.

The measurement selection criterion is to select a secondary measurement such that the averaging integrated error is minimized when one of the load disturbance (with $\tilde{\gamma} > 1$) is chosen as the nominal case. Therefore, the criterion can be summarized as follows:

1. Construct the Γ array.

Table 3. Averaging Integrated Error for Different Measurement Locations

Tray No.	Loads Variable				
	Propylene	n-Propane	Isobutane	1-Butene	n-Butane
14	0.1948	—	—	—	—
15	0.1952	—	0.2051	—	—
20	0.4658	—	0.5007	—	—

Table 4. Controller Parameters

	$K_{c,p}$	$\tau_{i,p}$	$K_{c,s}$
Single-Loop	284.4	11.37	—
(14,1)	2,655.7	7.11	−4.0
(20,1)	288.1	9.87	−1.223

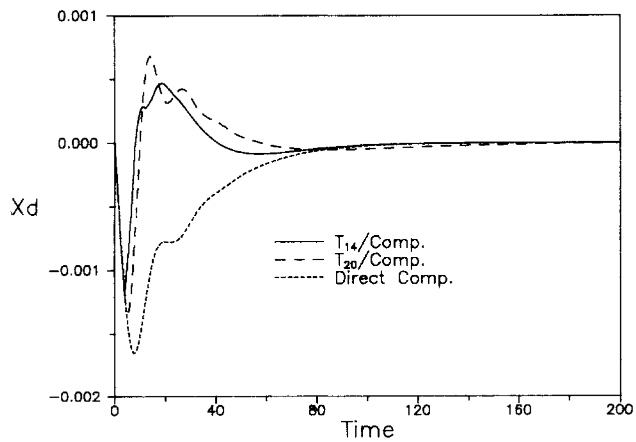

Figure 2. Closed-loop responses of T_{14}/comp., T_{20}/comp. and direct composition control for a step change in propylene feed composition.

2. Calculate $\overline{IE}_i^{(k)}$ for all measurement location (i's) with respect to all possible nominal load disturbance (all k's with $\gamma_{ik} > 1$).

3. Select the measurement location with the smallest averaging integrated error ($\overline{IE}_i^{(k)}$).

Application

A five-component, 20-tray depropanizer (Patke et al., 1982) is used to illustrate the effect of secondary variable to the performance of the parallel cascade control. The control objective is to maintain the overhead isobutane composition (Xd) constant. Table 1 gives process and load transfer functions for both primary (Xd) and secondary (T_i's) outputs. Patke et al. (1982) selected tray 20 for parallel cascade control. Following the proposed measurement selection criterion, the Γ array is given in Table 2. The $\overline{IE}$'s (Table 3) show trays 14 and 15 are better choices for placing the temperature sensors, e.g., when propylene feed composition changes is chosen as the nominal load variable (on which the design of K_s is based). PI and P-only controllers are used in the primary and secondary loops, respectively. The controllers are designed according to the procedure of Yu (1988) with minor modification. Table 4 gives controller parameters.

Comparison is made between single-loop control and indirect composition control with temperature control tray chosen as tray 14 or 20. Four minutes of analyzer dead time is added to the composition loop. Simulation results (Figure 2) show that indirect feedforward control performs better for the nominal load disturbance (a step change in the propylene feed composition). However, the performance of tray 20 parallel cascade control deteriorates as the load disturbance changes to the propane feed composition change (Figure 3). The Γ array in Table 2 predicts the result *a priori*. Indirect feedforward control with tray 14 temperature performs better in all disturbances studied. The results indicate that control quality of parallel cascade control can be improved by selecting appropriate temperature control tray.

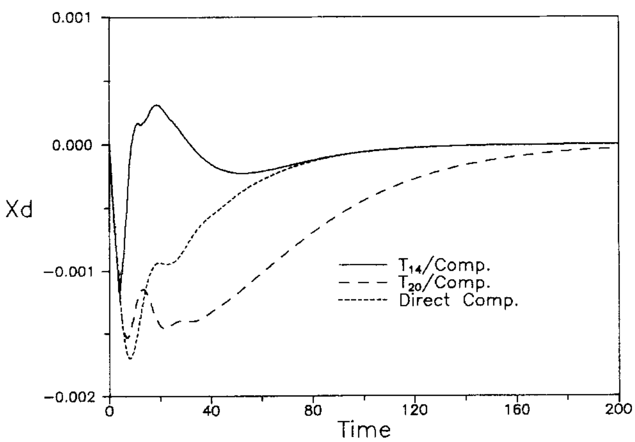

Figure 3. Closed-loop responses of T_{14}/comp., T_{20}/comp. and direct composition control for a step change in n-propane feed composition.

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Notation

IE = integrated error
$\overline{IE}$ = averaging integrated error
G = process transfer function
$\tilde{G}$ = nominal process transfer function
G_L = load transfer function
$\tilde{G}_L$ = nominal load transfer function
K = controller transfer function
K_c = controller gain
T = tray temperature
m = manipulated variable
x = controlled variable
Xd = composition of distillate

Greek letters

γ = interaction for load disturbance
$\tilde{\gamma}$ = interaction for the nominal load disturbance
Γ = interaction measure array
τ_i = reset time

Superscripts

k = kth load disturbance chosen as the nominal one
set = setpoint

Subscripts

ij = ith measurement location and jth load disturbance, respectively
CL = closed-loop
p = primary loop
PC = parallel cascade control
s = secondary loop
SL = single-loop control

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Appendix

Assume that PI controllers are used in the single-loop control and the primary loop of parallel cascade control and $IE_{PC} = IE_{SL}$ for a unit step set point change. For a load disturbance with an interaction measure γ (Eq. 4), the ratio of integrated error for a unit step load change becomes (Yu, 1988).

$$\frac{IE_{PC}}{IE_{SL}} = 1 - \frac{G_{s(0)}K_{s(0)}}{1 + G_{s(0)}K_{s(0)}}\gamma \tag{A1}$$

Note that Eq. A1 holds for all load disturbances. If the secondary controller is designed according to a nominal load variable, we have

$$K_s = \frac{1}{\left[\frac{\tilde{G}L_{s(0)} G_{p(0)}}{\tilde{G}L_{p(0)} G_{s(0)}} - 1\right] G_{s(0)}} \tag{A2}$$

Substituting Eq. A2 into Eq. A1, we have

$$\frac{IE_{PC}}{IE_{SL}} = \frac{\tilde{\gamma} - \gamma}{\tilde{\gamma}} \tag{A3}$$

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